Zero Product Property

$0 = x^2 + 7x + 12$

$0 = (x + 4)(x + 3)$

$x + 4 = 0$ $\quad x = -4$ $\qquad x + 3 = 0$ $\quad x = -3$

$0 = (x - 4)(2x + 5)(3x - 7)(x + 5)$

$x - 4 = 0$ $\quad x = 4$ $\qquad 2x + 5 = 0$ $\quad 2x = -5$ $\quad x = -\frac{5}{2}$ $\qquad 3x - 7 = 0$ $\quad 3x = 7$ $\quad x = \frac{7}{3}$ $\qquad x + 5 = 0$ $\quad x = -5$

Factor Theorem:

If f(k) = 0, [if the remainder is 0]
then x - k is a factor.

Ex. 1. Factor $f(x) = 2x^3 - 11x^2 + 3x + 36$

$f(x) = (x - 3)(\ ?\ )$

given that x - 3 is a factor

3	2	-11	3	36
	↓	6	-15	-36
	2	-5	-12	0

↑ Put a "3" outside

$2x^2 - 5x - 12$

$f(x) = (x - 3)(2x^2 - 5x - 12)$

$f(x) = (x - 3)(2x + 3)(x - 4)$

Factor Theorem Examples

If f(k) = 0, then (x - k) is a factor.

Factor: $f(x) = x^3 - 10x^2 + 19x + 30$ if $(x - 6)$ is a factor.

$f(x) = (x-6)(\quad ? \quad)$ ← Put a "6" outside

6	1	-10	19	30
	↓	6	-24	-30
	1	-4	-5	0

$f(x) = (x-6)(x^2-4x-5) = (x-6)(x-5)(x+1)$

Solve: $0 = x^3 + 6x^2 + 5x - 12$ if $(x + 4)$ is a factor.

$0 = (x+4)(\quad ? \quad)$ ← Put a "-4" outside.

-4	1	6	5	-12
	↓	-4	-8	12
	1	2	-3	0

$0 = (x+4)(x^2+2x-3) = (x+4)(x+3)(x-1)$

$x+4=0 \quad x+3=0 \quad x-1=0$

$x=-4 \quad x=-3 \quad x=1$

Find all the zeros: $f(x) = x^3 - 2x^2 - 21x - 18$ if -3 is a zero.

$f(x) = (x+3)(\quad ? \quad)$ ← $(x+3)$ is a factor [Put a "-3" outside]

-3	1	-2	-21	-18
	↓	-3	15	18
	1	-5	-6	0

$f(x) = (x+3)(x^2-5x-6)$

$0 = (x+3)(x-6)(x+1)$

$x+3=0 \quad x-6=0 \quad x+1=0$

$x=-3 \quad x=6 \quad x=-1$

Problems from the "Solving Polynomial Equations" Packet

2.) $(x^2+9)(x^2+5)=0$

$x^2+9=0$ $\quad$ $x^2+5=0$

$x^2=-9$ $\quad$ $x^2=-5$

$\sqrt{x^2}=\sqrt{-9}$ $\quad$ $\sqrt{x^2}=\sqrt{-5}$

$x=\pm\sqrt{-9}$ $\quad$ $x=\pm\sqrt{-5}$

$x=\pm i\cdot 3$ $\quad$ $x=\pm i\sqrt{5}$

$x=\pm 3i$

5.) $x^3-3x^2+2x=0$

$x(x^2-3x+2)=0$

$x(x-2)(x-1)=0$

$x=0$ $\quad$ $x-2=0$ $\quad$ $x-1=0$

$x=2$ $\quad$ $x=1$

13.) $5x^3+16x^2+13x+2=0$

$(x+2)(\quad ?\quad)=0$

-2 is a root. $(x+2)$ is a factor. Put a "-2" outside

-2 | 5 16 13 2

$\downarrow$ -10 -12 -2

5 6 1 0

$(x+2)(5x^2+6x+1)=0$

$(x+2)(5x+1)(x+1)=0$

$x+2=0$ $\quad$ $5x+1=0$ $\quad$ $x+1=0$

$x=-2$ $\quad$ $5x=-1$ $\quad$ $x=-1$

$x=-\frac{1}{5}$